The Usage of the K Factor in Heavy Ion Physics*

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It has been clear for many years that only a leading order, LO, evaluation of quantum chromodynamics, QCD, cross sections is inadequate to describe Drell-Yan, heavy quark, and jet production. For example, at LO lepton pairs from the Drell-Yan process and heavy quark pairs, $Q\overline{Q}$, are produced back-to-back with zero transverse momentum, p_T, of the pair. In addition, LO cross sections underestimate the measured cross sections.

Next-to-leading order, NLO, evaluations of such cross sections removed many of these inadequacies. Most modern parton distribution functions, PDFs, are evaluated at NLO and fewer new sets are evaluated at LO. Thus, full NLO calculations, using NLO PDFs, two-loop calculations of α_s and NLO fragmentation functions, where applicable, are preferable. However, only LO matrix elements are still employed for some processes such as NRQCD quarkonia production, not yet fully calculated at NLO. In addition, the LO matrix elements are inputs to event generators. Therefore, LO calculations are still of use, either for speed or ease of calculation, if normalized properly relative to NLO.

The K factor is used to normalize the LO calculations. In heavy ion physics, the K factor has often been assumed to be 2 without justification. These rather large K factors are usually multiplied by LO cross sections evaluated with NLO PDFs and the two-loop α_s. K should be determined according to a clearly defined prescription appropriate to the calculational method.

Before NLO calculations were generally available, the LO calculations were scaled up to the data by an arbitrary factor, the original K factor,

$$K_{\rm exp} = \frac{\sigma_{\rm exp}}{\sigma_{\rm th}} \; . \tag{1}$$

Higher order calculations are compared to data by making the ratio 'data/theory'. If this ratio is not unity everywhere, it is similar to $K_{\rm exp}$. The K factor is more often determined theoretically. The usual theoretical K factor is

$$K_{\rm th}^{(1)} = \frac{\sigma_{\rm NLO}}{\sigma_{\rm LO}} \tag{2}$$

where the superscript '(1)' refers to the fact that only the first order correction is included. When further higher order corrections exist, higher order theoretical K factors can also be defined.

There is some ambiguity in $K_{\rm th}^{(1)}$. The NLO contribution, $\sigma(\alpha_s^{n+1})$ where n is the power of α_s at LO, is calculated with α_s evaluated to two loops and with NLO PDFs. However, the LO cross section can be calculated in more than one way. If LO PDFs and fragmentation functions as well as the one-loop α_s is used (the most appropriate), then

$$\begin{aligned} \sigma_{\rm LO(1)} &\equiv \sigma_1(\alpha_s^n) \; , & (3)\\ \sigma_{\rm NLO(1)} &\equiv \sigma_1(\alpha_s^n) + \sigma(\alpha_s^{n+1}) \; . & (4) \end{aligned}$$

On the other hand, if the LO cross section is calculated with NLO PDFs and the two-loop α_s (most typical),

$$\begin{aligned} \sigma_{\rm LO(2)} &\equiv \sigma_2(\alpha_s^n) \; , & (5)\\ \sigma_{\rm NLO(2)} &\equiv \sigma_2(\alpha_s^n) + \sigma(\alpha_s^{n+1}) \; . & (6) \end{aligned}$$

Thus, in principle, $K_{\rm th}^{(1)}$ could be formed several ways:

$$K_{\rm th,0}^{(1)} \equiv \frac{\sigma_{\rm NLO(2)}}{\sigma_{\rm LO(2)}} \quad K_{\rm th,1}^{(1)} \equiv \frac{\sigma_{\rm NLO(1)}}{\sigma_{\rm LO(1)}} \quad K_{\rm th,2}^{(1)} \equiv \frac{\sigma_{\rm NLO(2)}}{\sigma_{\rm LO(1)}} \; . \tag{7}$$

In our calculations, we investigated the differences between $K_{\rm th,0}^{(1)}$, $K_{\rm th,1}^{(1)}$ and $K_{\rm th,2}^{(1)}$.

We found that the K factor is in general not constant for a given distribution. The Drell-Yan K factor is not constant with mass, increasing at low masses and also changes at high masses. The rapidity range over which K is constant at the SPS decreases with increasing mass as the edge of phase space is approached at lower rapidities. The K factors for heavy quarks are most strongly dependent on p_T since the NLO corrections are large for $p_T \geq m_Q$.

Clearly a constant K factor is inappropriate for all kinematic variables. Indeed it depends not only on the process but also on the calculated distribution. Therefore, to best utilize knowledge of the NLO and higher order corrections, the differential K factor should be determined for the kinematic variables of interest such as p_T, y or M. Whatever choice is made, it is important that the determination of $K_{\rm th}$ be clearly described and applied consistently.

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